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INITIAL-STATE COLOUR DIPOLE EMISSION ASSOCIATED WITH QCD POMERON EXCHANGE

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ABSTRACT

The initial-state radiation of soft colour dipoles produced together with a single QCD Pomeron exchange (BFKL) in onium-onium scattering is calculated in the framework of Mueller's approach. The resulting dipole production grows with increasing energy and reveals an unexpected feature of a power-law tail at appreciably large transverse distances from the collision axis, this phenomenon being related to the scale-invariant structure of dipole-dipole correlations.

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High energy onium-onium scattering is a simple process which can be used to study the physics of the perturbative QCD Pomeron, the so-called BFKL singularity[1]. Recently a quantitative picture in which a high-Q^2 $q\bar{q}$ (or *onium*) state looks like a collection of colour dipoles of various sizes has been developped by Mueller[2−4] . The QCD Pomeron elastic amplitude is recovered in this dipole picture provided the onium-onium elastic scattering comes from a dipole in one onium state scattering off a dipole in the other onium state by means of two-gluon exchange[2−4,5].In this paper we elaborate some consequences of the dipole approach for the initial-state radiation associated with a single Pomeron exchange.

Our starting point is the observation that the onium-onium scattering process is accompanied by a radiation due to colour dipoles which – while present in the initial state – are released during the collision. In this note we present an explicit computation of this dipole emission process following Mueller's approach, and more specifically that of Ref.[4]. We have found that the cross-section for the production of dipoles emitted from one of the colliding onia can be approximated by:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}^2r} = \int \mathrm{d}^2x_{10}\mathrm{d}^2x'_{10} \int_0^1 \mathrm{d}z\ \mathrm{d}z'\ \phi\left(x_{10}, z\right) \phi\left(x'_{10}, z'\right) \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}^2r},$$

where

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}r^2} = \sigma_{\mathrm{tot}}\ \frac{1}{xr^2}\ \mathrm{e}^{\left((\alpha_p-1)\frac{Y}{2}\right)}\ \frac{x_{10}}{x} \left(\frac{r}{x_{10}}\right)^{\gamma_M} \varphi\left(\frac{xx_{10}}{r^2}\right)\ , \tag{1}$$

where[4] $\sigma_{\mathrm{tot}} = 2\pi\ x_{10}x'_{10}\ \alpha^2\ \mathrm{e}^{(\alpha_p-1)\ Y}\ \left(\frac{2a}{\pi}\right)^{1/2}$, $\alpha_p = 1 + \frac{\alpha N_c}{\pi}\ 4\ln 2$ is the intercept of the QCD Pomeron singularity (BFKL) and φ is a function which acts as a cut-off on the scaling behaviour $\left(\frac{r}{x_{10}}\right)^{\gamma_M}$. Within a high-energy approximation, this function can be parametrized as follows:

$$\varphi \approx \mathcal{C}\ \left(\frac{2a}{\pi}\right)^{3/2}\ \ln\left(\frac{r^2}{xx_{10}}\right) \exp\left(-a\ \ln^2\left(\frac{r^2}{xx_{10}}\right)\right)\ , \tag{2}$$

where $\mathcal{C}$ is a constant and the cut-off scale is defined as:

$$a \equiv a(Y) = \left[7\alpha\ N_c\zeta(3)Y/\pi\right]^{-1}\ \approx\ \left[3\left(\alpha_p-1\right)Y\right]^{-1} \tag{3}$$

In our result (1), and following the notations of Refs.[2-4], $\phi\left(x_{10}, z\right)$ defines the square of the heavy $q\bar{q}$ component of the onium wave-function with transverse-coordinate separation x_{10} and light-cone momentum z of the antiquark with respect to the onium; x is

the transverse size of the emitted dipole, $\vec{r}$ its transverse coordinate and $e^{-Y/2} = \frac{P_+}{E_{c.m.}}$ denotes the light-cone momentum fraction of the softest gluon involved in constituting the emitted dipole. As will be explained further on, the important parameter $\gamma_M \approx .37$ is a solution[4] of the equation

$$\chi\left(\gamma_M\right) \equiv 2\psi(1) - \psi\left(1-\gamma_M/2\right) - \psi\left(\gamma_M/2\right) = 2\ \chi(1), \tag{4}$$

where $\psi(\gamma) \equiv \frac{\mathrm{d}}{\mathrm{d}\gamma}\ \ln\Gamma,\ \chi(1)\ \equiv\ 4\ln 2$. The formula (1) is expected to be valid provided

$$r \gg x, x_{10}\ ; \qquad 0 < \ln\frac{r}{x}, \ln\frac{r^2}{x_{10}x}\ <\ a^{-1/2}(Y)\ , \tag{5}$$

requiring large distances with respect to dipole sizes while limited by the total amount of c.m.s. energy.

Before we proceed to outline the derivation of Eqs.(1-3), let us point out their two interesting features.

(a) for fixed x and r the density of emitted dipoles increases as a power of the incident energy

$$\frac{1}{\sigma_{\rm tot}}\ \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}^2r} \sim \left(\frac{E_{c.m.}}{P_+}\right)^{\alpha_p-1} \equiv \left(\frac{E_{c.m.}}{P_+}\right)^{\chi(1)\ \frac{\alpha N_c}{\pi}}$$

with a power fully determined by the QCD Pomeron intercept α_p. Thus, the contribution of soft-gluon radiation is increasing with energy and very likely dominant at high enough energy at least in the region where the QCD Pomeron exchange is relevant. Note that this contributions increases also when P_+ decreases.

(b) For fixed dipole size x the distribution reveals a *power-law* tail in the transverse position coordinate r, namely

$$xr^2\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}^2r} \sim\ r^{\gamma_M}\ , \tag{6}$$

which is expected to extend until a distance cut-off defined by Eqs.(2,3), that is:

$$r^2 \leq r^2_{\rm max} = x_{10}x\ \mathrm{e}^{a^{-1/2}} \approx x_{10}x\ \exp\left(\sqrt{3(\alpha_p-1)Y}\right) \tag{7}$$

One sees that $r_{\rm max}$ increases as the exponential of $\sqrt{\ln s}$. Thus – at high enough energies – the power-law behaviour (6) is valid up to distances appreciably exceeding the initial

(x_{10}) and emitted (x) dipole sizes. Since other contributions to gluon emission are not expected to possess this large distance tail, we infer that the power-law tail (6) should be a dominant component of gluon emission at relatively large distances from the collision axis.

2. Let us now outline the derivation of Eqs.(1-4). As already explained, we intend to estimate the emission of colour dipoles from the initial-state. To this end, we first write the formula for the inclusive cross-section for emission of a dipole from one of the colliding onia. It reads:

$$x \, \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x \mathrm{d}r^2} = 4\pi\alpha^2 \int_0^\infty \frac{\mathrm{d}l}{l^3} \, \frac{\mathrm{d}\bar{x}}{\bar{x}} \, \frac{\mathrm{d}\bar{x}'}{\bar{x}'} \left[1 - J_0\left(l\bar{x}\right)\right] \left[1 - J_0\left(l\bar{x}'\right)\right] \\ \times \int \mathrm{d}^2 b \; \mathrm{d}^2 b' \; n^{(1)} \left(x'_{01}, \vec{b}' - \vec{b}, \bar{x}'; \frac{Y}{2}\right) n^{(2)} \left(x_{01}, \vec{b}, \bar{x}; \vec{r}, x; \frac{Y}{2}\right) \tag{8}$$

where $n^{(1)}$ is the *single* dipole density in the onium of size x'_{01} and $n^{(2)}$ is the *double* dipole density in the onium of size x_{01}.

This formula -which is an extension of Mueller's formula for the total onium-onium cross-section (see Eq.(3) of Ref.[4])- expresses simply the fact that the number of emitted dipoles is just the number of dipoles present in the initial state *whenever* the interaction took place. The process is illustrated in Figure 1 where the geometry of the reaction in the plane transverse to the collision axis is represented. The important feature of Eq.(8) is that it contains the double-dipole density in one of the initial onia (since one of the two dipoles is involved in the interaction mechanism). Thus, it is sensitive to *correlations* between dipoles in the same onium state. As we shall see, this has non-trivial consequences. In fact, already at this stage, one may expect that these correlations should be rather strong because, as shown in Refs.[2-4], the colour dipoles which contribute to the onium wave function are formed in a cascade process and thus cannot be independent. Furthermore, since this cascade is scale-invariant one expects also scale invariance in the dipole-dipole correlations which -in turn- is likely to be a reflection of the inner conformal invariance, known to be rooted in the formalism of the BFKL Pomeron[8].

The explicit expression for $n^{(1)}$, was given in Ref.[4]:

$$n^{(1)}\left(x'_{01},\vec{b}'-\vec{b},\bar{x}';Y/2\right)\equiv\frac{x'_{01}}{4x'\left|\vec{b}'-\vec{b}\right|^2}\ln\frac{\left|\vec{b}'-\vec{b}\right|^2}{x'_{01}\bar{x}'}\exp\left(-a\ \ln^2\frac{\left|\vec{b}'-\vec{b}\right|^2}{x'_{01}\bar{x}'}\right)\times$$
$$\times\exp\left[\left(\alpha_{p-1}\right)Y/2\right]\left(\frac{4a}{\pi}\right)^{-3/2}, \qquad (9)$$

where $a=a(Y)$ is defined as in Eq.(3).

The explicit expression for $n^{(2)}$ is not yet known, besides an approximate formula[4] which is insufficient for our purpose of deriving the overall behaviour of $\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\ \mathrm{d}^2r}$ as a function of the different variables of the problem. To obtain an improved formula, valid at large distances from the colliding onia, we employ an approximate form of the equation for $n^{(2)}$ (given in [4]), with only terms dominant at large distances being kept. Written for the Mellin-Transform $\tilde{n}$ of $n^{(2)}$ it reads:

$$\frac{\partial\tilde{n}^{(2)}}{\partial Y}\left(\gamma,\vec{b},\bar{x};\vec{r},x;Y\right)=\frac{\alpha N_c}{\pi}\chi(\gamma)\ \tilde{n}^{(2)}\left(\gamma,\vec{b};\vec{r},x;Y\right) \qquad (10)$$
$$+\frac{\alpha N_c}{\pi^2}\int x_{10}^{1-\gamma}\mathrm{d}x_{10}\int\frac{\mathrm{d}^2x_2}{x_{12}^2x_{02}^2}n^{(1)}\left(x_{02},\vec{b},\bar{x};Y\right)n^{(1)}\left(x_{12},\vec{r},x;Y\right)$$

where $\chi(\gamma)$ is defined as in (4). $n^{(2)}$ may be recovered from the solution of (10) by writing

$$n^{(2)}\left(x_{01},\vec{b},\bar{x};\vec{r},x;Y\right)=\int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}\gamma}{2i\pi}\ x_{10}^{\gamma}\ \tilde{n}^{(2)}\left(\gamma,\vec{b},\bar{x};\vec{r},x;Y\right)\ . \qquad (11)$$

with the real constant $c<2$ for convergence condition, and to the right (in the complex $\gamma-$plane) of all singularities of $\tilde{n}^{(2)}$.

Let us first find an expression for the inhomogeneous term of Eq.(10) denoted $\tilde{n}_0^{(2)}$. Using the analytic form (9) for the single-dipole distributions n_1 and the known Jacobian of the transformation of variables from $\vec{x}_2$ to x_{02}, x_{12}, namely[2]

$$\mathrm{d}^2x_2\equiv 2\pi\ \mathrm{d}x_{02}\mathrm{d}x_{12}\ x_{02}x_{12}\int_0^\infty p\mathrm{d}p\ J_0\left(px_{10}\right)J_0\left(px_{12}\right)J_0\left(px_{02}\right)\ , \qquad (12)$$

one may write after some algebraic manipulations:

$$\tilde{n}_0^{(2)}\left(\gamma,\vec{b},\bar{x};\bar{r},x;Y\right)=\exp\left[2\left(\alpha_p-1\right)Y\right]\times\mathcal{G}\left(\gamma,\vec{b},\bar{x};\bar{r},x;Y\right) \qquad (13)$$
$$\mathcal{G}\left(\gamma,\vec{b},\bar{x};\bar{r},x;Y\right)=2\frac{\alpha N_c}{\pi}\left(\frac{2a}{\pi}\right)^3\frac{1}{16\left(\bar{x}b^2\right)\left(xr^2\right)}\int\int\mathrm{d}x_{02}\mathrm{d}x_{12}\ \ln\left(\frac{b^2}{x_{12}\bar{x}}\right)\ln\left(\frac{r^2}{x_{02}x}\right)$$
$$\times\ \exp\left\{-\frac{a}{2}\left(\ln^2\frac{b^2}{x_{12}\bar{x}}+\ln^2\frac{r^2}{x_{02}x}\right)\right\}W\left(x_{02},x_{12}\right).$$

The function $W\left(x_{01}, x_{02}\right)$ can be expressed in terms of generalized hypergeometric series as follows:

$$
\begin{aligned}
W\left(x_{02}, x_{12}\right) &\equiv \int_0^{\infty} x_{10}^{1-\gamma} \mathrm{d}x_{10}\ p\mathrm{d}p\ J_0\left(p x_{10}\right) J_0\left(p x_{12}\right) J_0\left(p x_{02}\right) \\
&= \theta\left(x_{02}-x_{12}\right)\ x_{02}^{-\gamma}\ {}_2F_1\left[\left(\frac{x_{12}}{x_{02}}\right)^2\right]+\{1 \longleftrightarrow 2\}\ , \qquad (14)
\end{aligned}
$$

where we used a shortened notation for the hypergeometric function[7]

$$
{}_2F_1\left[y^2\right] \equiv\ {}_2F_1\left(\frac{\gamma}{2}, \frac{\gamma}{2}; 1; y^2\right)\ .
$$

From formula (13), it is quite clear that the dominant energy dependence of $\tilde{n}_0$ is given by the exponential term $\exp\left\{2\left(\alpha_p-1\right)\right\}$. Consequently, we can approximately solve Eq.(10) for $\tilde{n}^{(2)}$ and obtain (in the limit $Y \longrightarrow \infty$)

$$
\tilde{n}^{(2)}(\gamma, Y) \approx \mathrm{e}^{\frac{\alpha N_c}{\pi}\chi(\gamma) Y}\left(2\left(\alpha_p-1\right)-\frac{\alpha N_c}{\pi}\chi(\gamma)\right)^{-1} \times\ \mathcal{G}\ . \qquad (15)
$$

Using (15) we can calculate the Mellin transform of $\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}r^2}$, i.e.

$$
\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x\mathrm{d}r^2}\ (\gamma)\ = \int\ \mathrm{d}x_{10}^{-\gamma-1}\ \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}r^2}\ \left(x_{10}\right) \qquad (16)
$$

Indeed, inserting (8) and (13) into (16), it is possible[4] to integrate over $\mathrm{d}l$, $\mathrm{d}\bar{x}$, $\mathrm{d}\bar{x}'$, $\mathrm{d}^2 b$, $\mathrm{d}^2 b'$ since they decouple due to factorizability properties of $\tilde{n}_0^{(2)}$. All in all, one gets

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x\mathrm{d}r^2} &= \hat{\sigma}_{\mathrm{tot}} \times\ \sqrt{2}\ \frac{\alpha N_c}{\pi}\ \frac{1}{x r^2}\ \frac{\exp\left[\left(\frac{\alpha N_c}{\pi}\chi(\gamma)-\left(\alpha_p-1\right)\right)\ Y/2\right]}{2\ \left(\alpha_p-1\right)\ -\ \frac{\alpha N_c}{\pi}\chi(\gamma)}\left(\frac{2a}{\pi}\right)^{3/2} \qquad (17) \\
&\times \int_0^r \int_0^r\ \mathrm{d}x_{12}\mathrm{d}x_{02}\ \ln\frac{r^2}{x_{12}x}\ \mathrm{e}^{-a\ln^2\left(\frac{r^2}{x_{12}x}\right)}\ W\left(x_{02}, x_{12}\right)\ ,
\end{aligned}
$$

where[4] $\hat{\sigma}_{\mathrm{tot}}=\frac{\sigma_{\mathrm{tot}}}{x_{10}}=2\pi x_{10}'\ \alpha^2\ \mathrm{e}^{\left(\alpha_p-1\right)\ Y}\ \left(\frac{2\ a}{\pi}\right)^{1/2}$.

With the explicit form of $W\left(x_{02}, x_{12}\right)$, Eq.(14), the final integration over x_{02} and x_{12} can be performed. Using the known asymptotic expansion of the error function[7] within the approximations defined in (5), one obtains as a final result:

$$
\begin{aligned}
\frac{\mathrm{d}\tilde{\sigma}}{\mathrm{d}x\mathrm{d}r^2} &= \hat{\sigma}_{\mathrm{tot}}(x_{10}')\ \mathcal{H}(\gamma)\ \sqrt{2}\ \frac{\alpha N_c}{\pi}\left(\frac{2a}{\pi}\right)^{3/2} \\
&\times\ \frac{r^{-\gamma}}{x}\ \frac{\exp\left[\left(\frac{\alpha N_c}{\pi}\chi(\gamma)-\left(\alpha_p-1\right)\right)\ Y/2\right]}{2\ \left(\alpha_p-1\right)\ -\ \frac{\alpha N_c}{\pi}\chi(\gamma)} \qquad (18) \\
&\frac{\times\ \ln\left(\frac{r}{x}\right)\ \exp\left[-a\ln^2\left(\frac{r}{x}\right)\right]}{2+2a\ln\left(\frac{r}{x}\right)-\ \gamma}
\end{aligned}
$$

with

$$\mathcal{H}(\gamma) = \int_0^1 \mathrm{d}y \; {}_2F_1(y^2) \left(1 + y^{\gamma-2}\right).$$

To obtain $\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}x\mathrm{d}r^2}$, the inverse Mellin transform of (18) has to be determined. In performing it, one has to take into account the poles in the $\gamma-$variable present in the denominators of expression (18). In fact the one which is less but closest to the $2 \pm i\infty$ line in the complex γ-plane will dominate. One obtains:

$$\frac{\alpha N_c}{\pi} \, \chi\left(\gamma^*\right) = 2(\alpha_p \; - \; 1) \equiv \; \frac{2 \; \alpha N_c}{\pi} \; \chi(1),$$

Denoting for convenience the solution $\gamma^* = 2 - \gamma_M$, and using the explicit form of the kernel $\chi(\gamma)$ leads to equation (4) for γ_M. Notice that the other denominator in (18) corresponds to a pole outside the integration contour (at $\gamma > 2$). In the same way, among the different solutions of Eq.(4) we justify the physical choice made in Ref.4, namely the choice of the pole nearest and below $\gamma = 2$.

Using the relation:

$$r^{-\gamma} \; \frac{\ln\left(\frac{r}{x}\right) \; \mathrm{e}^{-a\ln^2\left(\frac{r}{x}\right)}}{\gamma^* - \gamma} \equiv \int_0^r x_{10}^{-\gamma-1} \; \mathrm{d}x_{10} \left(\frac{x_{10}}{r}\right)^{\gamma^* - 2a\ln\left(\frac{r}{x}\right)} \times$$

$$\times \ln\left(\frac{r^2}{x x_{10}}\right) \; \mathrm{e}^{-a\ln^2\left(\frac{r^2}{x x_{10}}\right)}, \tag{19}$$

valid for $\frac{r}{x} \gg 1$, we obtain formula (1) as the inverse Mellin transform reciprocal to (19). The constant $\mathcal{C}$ in (1) is given by

$$\mathcal{C} = \frac{\sqrt{2} \; \mathcal{H}(\gamma^*) \;\; (2 - \gamma *)^{-1}}{\psi'(1 - \gamma^*/2) - \psi'(\gamma^*/2)}. \tag{20}$$

It is important to notice that the integration range $0 \to r$ in (19) is required by the physical constraint $x_{10} \ll r$ on the cross-section.[1] This constraint is instrumental in the determination of the appropriate cut-off $\ln^2\left(\frac{r^2}{x x_{10}}\right) \; < \; a^{-1}$.

3. The most interesting feature of the cross-section expressions (1)-(3) is the factor r^{γ_M}, responsible for the non-trivial power-law behaviour at rather large r (at least for

[1] The approximation $a \ln\left(\frac{r}{x}\right) \ll 1$ has been used in the derivation of (1) from (18) and (20). A more complete treatment is possible but is not relevant for the discussion which always assumes the approximation scheme (5).

large enough incident energy). This factor is a direct consequence of the fact[4] that the two-dipole density $n^{(2)}$ in an onium state is not a mere direct product of two single dipole densities $n^{(1)} \otimes n^{(1)}$. In other words, it is a consequence of the scale-invariant *correlations* between colour dipoles located at different positions in the transverse space of the onium. These correlations are – in turn – to be considered as consequences of the (self-similar) cascading nature of dipole emission, as explicited in Mueller's approach[2−4].

Since $\gamma_M > 0$, the power-law tail in (1)-(3) is not integrable and thus the physical distribution becomes integrable only because of the cut-off $r < r_{\max}$, see (7), (22). Consequently, the Fourier transform of $\frac{d\hat{\sigma}}{d^2r}$ reveals a power-law behaviour at small transverse-momentum distance when it is greater than $r_{\max}^{-1}$. This feature gives interesting possibilities of observing this component of soft gluon emission either in the form of a power-law "spike" at very small transverse momenta of the produced hadrons or as the power-law "singularity" in HBT correlations between identical hadrons[2]. One may even hope that the value of γ_M can then be measured and confronted with theory. However, the feasibility of this program depends - at least to some extent - on the mechanism of the hadronization of the colour dipoles and therefore its discussion goes beyond the scope of the present paper. In particular a thorough discussion of the rôle of the dipole size x is needed. Clearly further work and other physical tests of the predicted power law tail (before the cut-off) is deserved, especially in the experimental context of deep inelastic scattering at HERA, where conditions approaching onium-onium scattering can be realized.

Finally, it is not excluded that effects similar to the ones computed in the theoretical framework of onium-onium scattering, could be present in hadronic collisions at high-energy. Obviously these processes are expected to be dynamically dominated by (yet uncalculable) non-perturbative contributions. However, if QCD Pomeron exchange (BFKL) is responsible for even a small part of the total hadron-hadron cross-section, the soft gluon emission of the type described here can be the only contribution to the production process extending up to rather large transverse distances. If this is the case, it may

[2] For incoherent emission, the HBT correlations between momenta of identical particles are approximately given by the square of the Fourier-transformed source density in space[9]. Therefore the power-law of the space density implies the power-law singularity at small momentum difference[10].

be possible to observe small but clear effects related to the power-law behaviour computed from perturbative QCD. In this context, it is worth mentioning that the existence of a perturbative QCD tail of the hadronic wave-function is advocated in some other cases, such the proton form-factor at high Q^2 or the colour transparency effect in reactions involving nuclei[11]. However, the limitations due to confinement forces must be better understood before definite predictions can be formulated.

In conclusion, we have computed the emission of colour dipoles induced by QCD pomeron exchange in onium-onium scattering. The resulting emission increases as a power of the ratio of the center-of-mass energy over the light-cone momentum of the softest gluon of the emitted dipole. It shows the interesting feature of a power-law tail in the transverse distance with respect to the collision axis, reflecting the conformal invariance of the underlying theory.

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[6] See Eq.(A.1) in Ref.[4]. We have considered the Y-differential version of this equation, which is easily shown to be identical to the integrated version of Ref.(4). This equivalence can be traced back to the equation defining the generating function of multi-dipole distributions.

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Figure Caption

Fig.1 : Transverse-Plane Geometry of Dipole-Emission.

The dashed area around each dipole is of a typical size of the order of the associated dipole length. They are assumed to be much smaller than the ranges in $\vec{r}$, $\vec{b}$, $\vec{b}'$, $\vec{b}' - \vec{b}$.